Hyperbola

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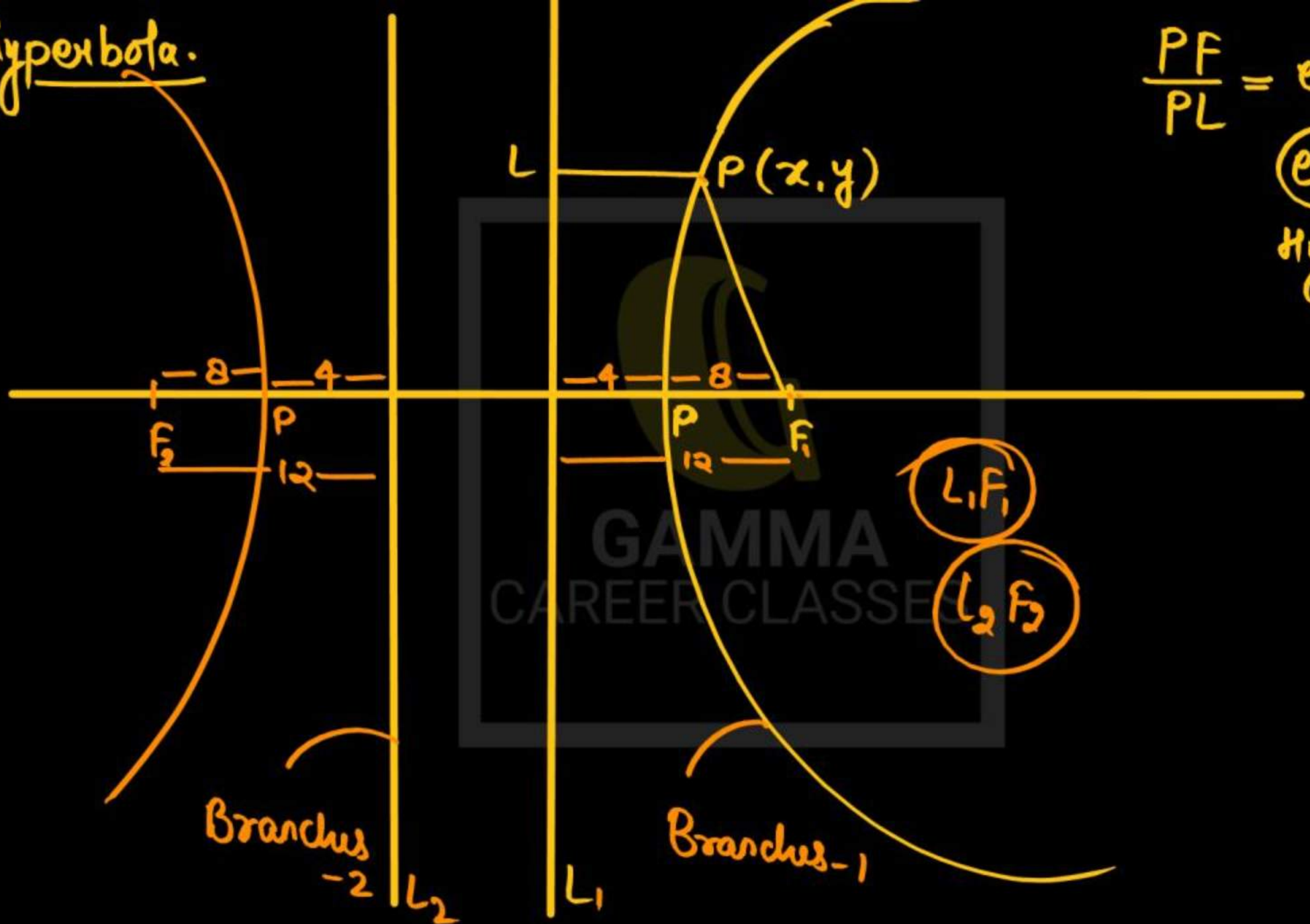
Hyperbola.



$\frac{PF}{PL} = e$

$e < 1$ ellipse

Hyperbola

$e > 1$

$\frac{PF}{PL} = \frac{2}{1}$

$\frac{8}{4} = 2$

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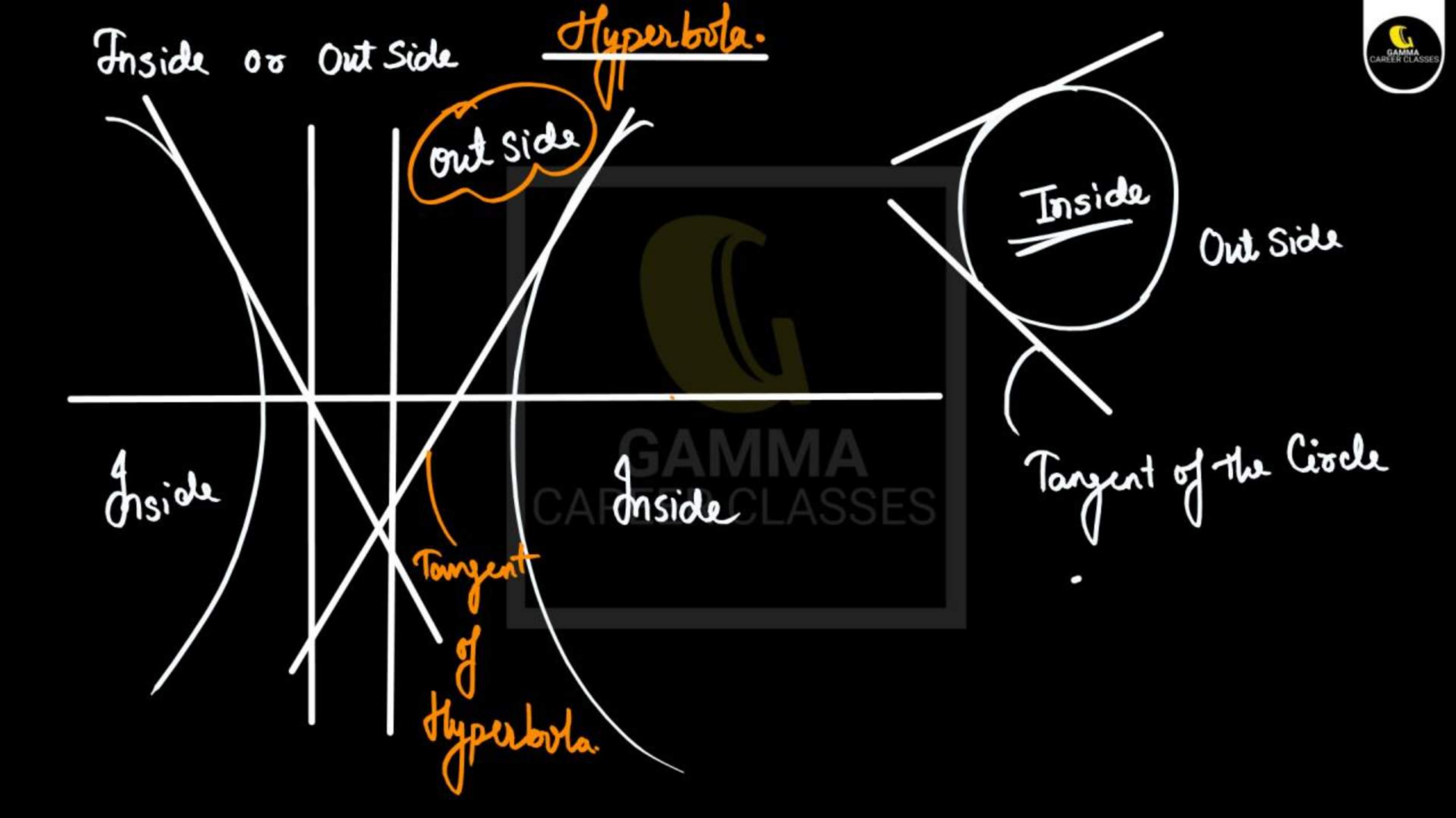
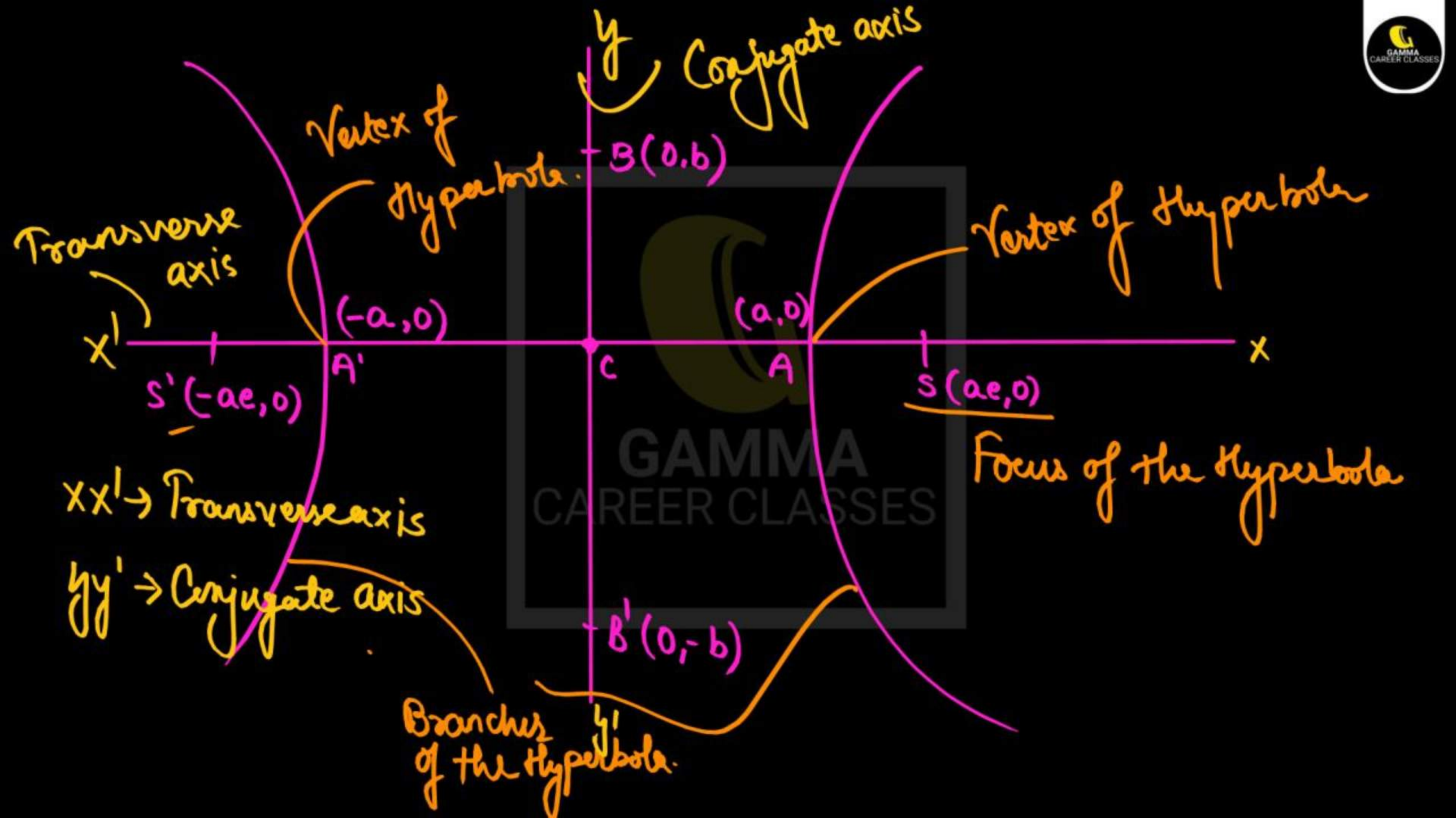
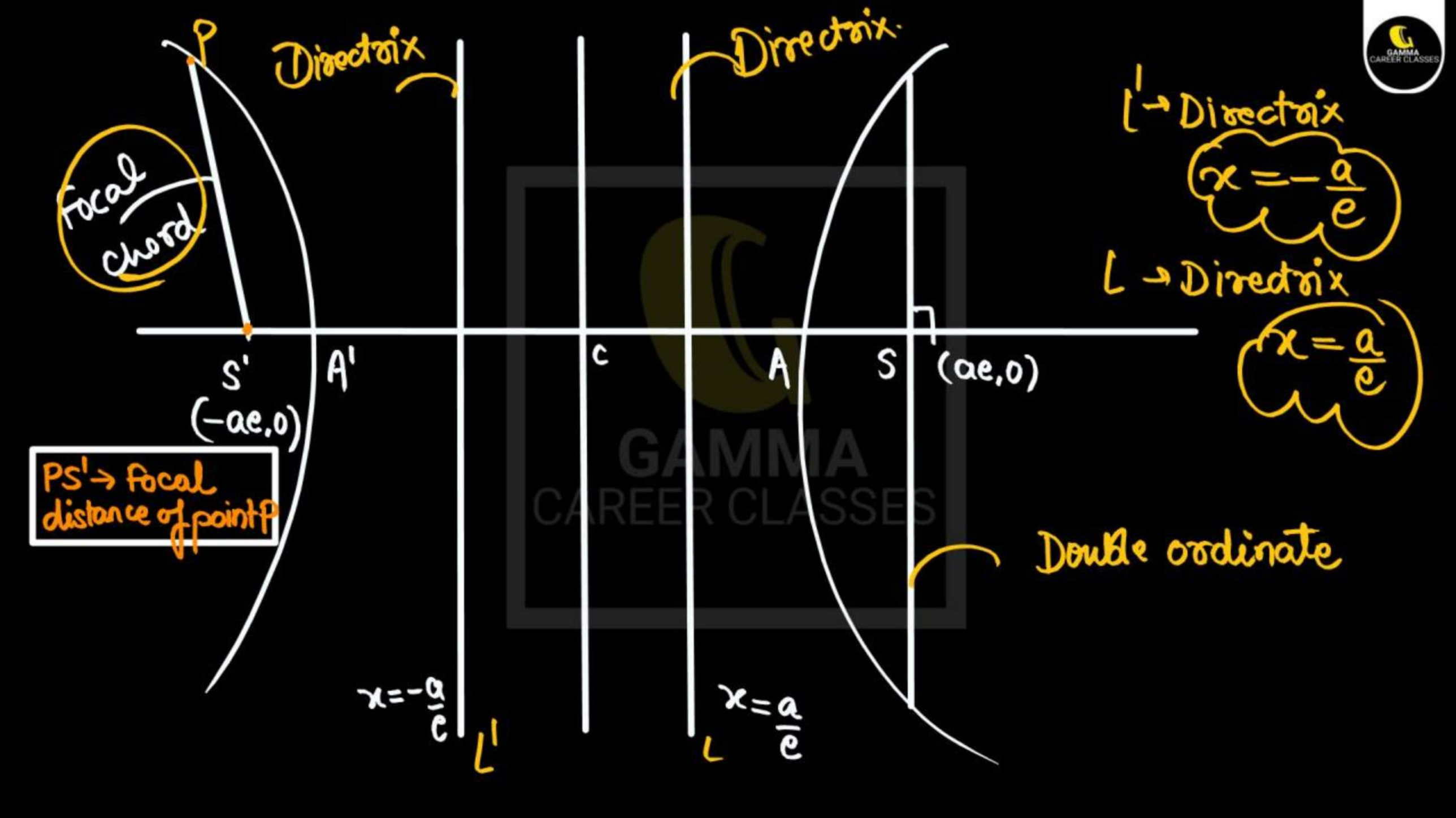

Inside or Out Side
Hyperbola.
Out Side
Inside
Inside
Tangent of Hyperbola.
Inside
Out Side
Tangent of the Circle


y
Conjugate axis
Vertex of Hyperbola
B(0,b)
Transverse axis
Vertex of Hyperbola
x'
(-a,0)
(a,0)
x
A'
C
A
S'(-ae,0)
S(ae,0)
Focus of the Hyperbola
XX' → Transverse axis
YY' → Conjugate axis
B'(0,-b)
Branches of the Hyperbola.
y'


P
Directrix
Directrix
Focal chord
S'
(−ae,0)
A'
C
A
S
(ae,0)
PS' → focal distance of point P
x = −a/e
L'
x = a/e
L
L' → Directrix
x = −a/e
L → Directrix
x = a/e
Double ordinate
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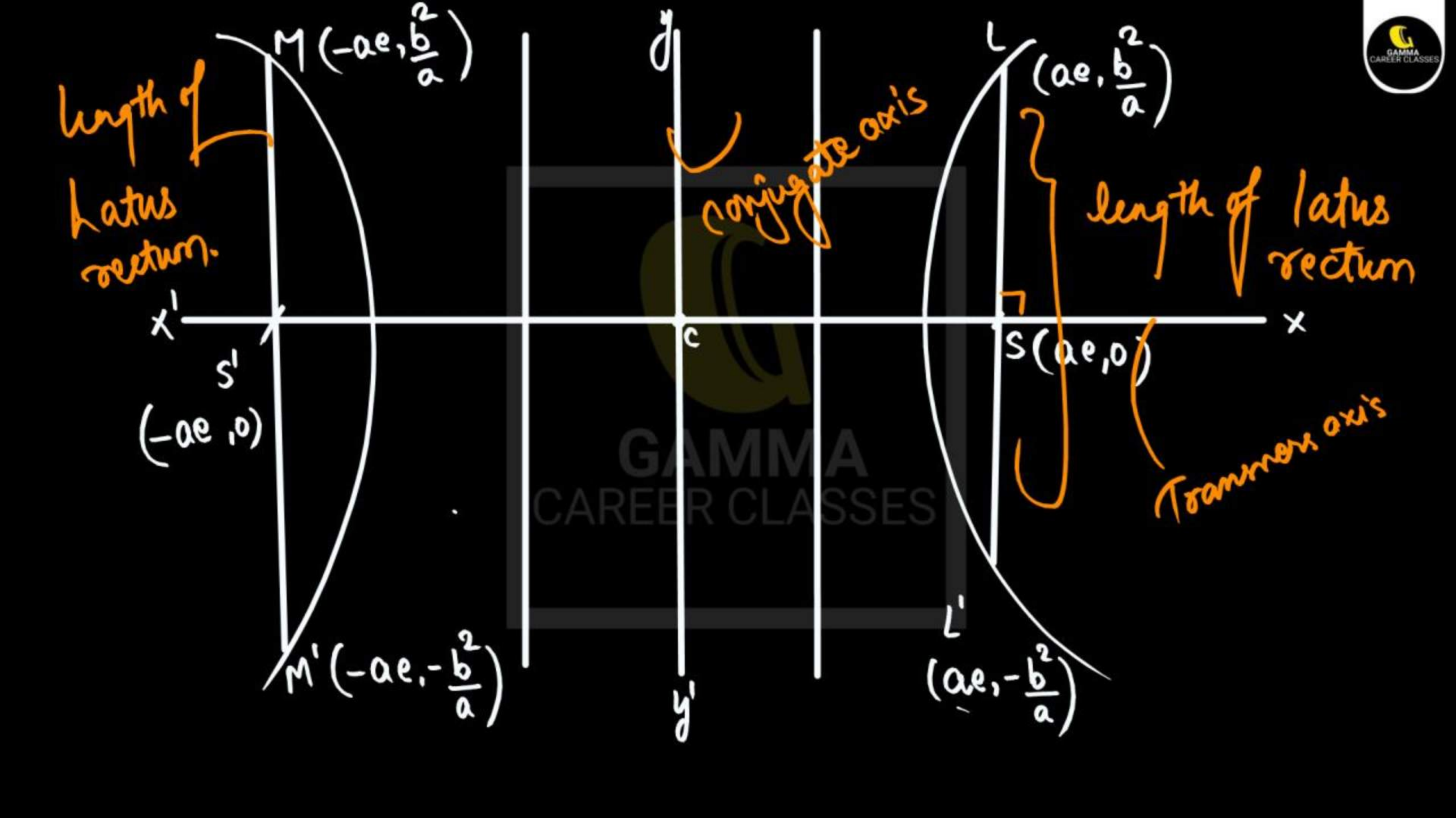

M $(-ae, \frac{b^2}{a})$
Length of latus rectum.
y
conjugate axis
L
$(ae, \frac{b^2}{a})$
length of latus rectum
x'
S'
$(-ae, 0)$
C
S $(ae, 0)$
x
Transverse axis
M' $(-ae, -\frac{b^2}{a})$
y'
L'
$(ae, -\frac{b^2}{a})$

Equation of the Hyperbola in standard form

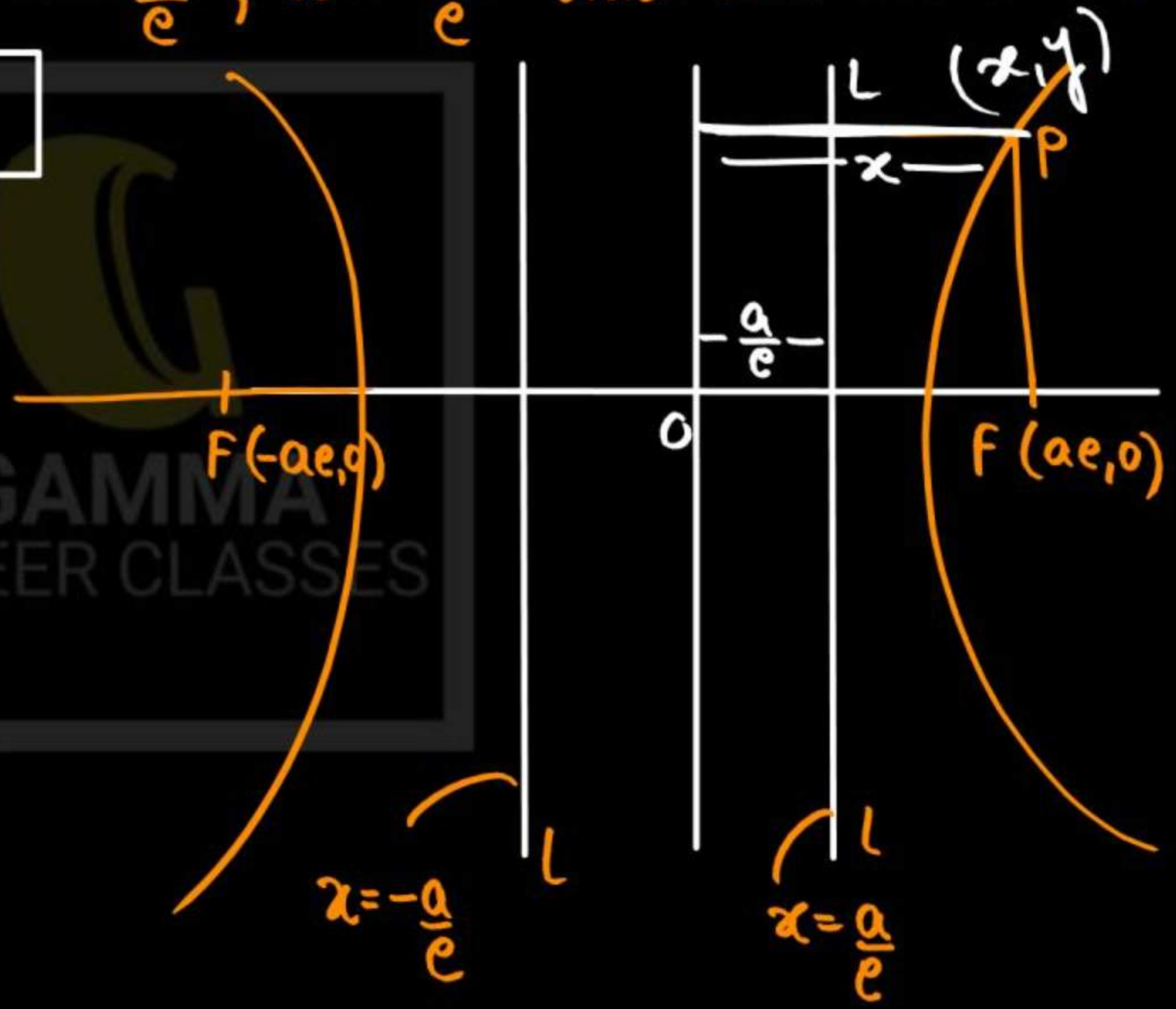
A Hyperbola of a directrix $x = \frac{a}{e}$, $x = -\frac{a}{e}$ and foci are $(ae, 0)$ $(-ae, 0)$, $P(x, y)$

$$\boxed{\frac{PF}{PL} = e}$$

$$\frac{\sqrt{(x-ae)^2 + (y-0)^2}}{\left|x - \frac{a}{e}\right|} = e$$

Sq. Both Side

$$(x-ae)^2 + y^2 = e^2\left(x - \frac{a}{e}\right)^2$$



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$$(x-ae)^2+y^2 = e^2\left(x-\frac{a}{e}\right)^2$$

$$x^2+a^2e^2-2aex+y^2 = e^2\left[x^2+\frac{a^2}{e^2}-2x\times\frac{a}{e}\right]$$

$$x^2+a^2e^2-2aex+y^2 = e^2x^2+\frac{a^2xe^2}{e^2}-\frac{2axe^2}{e}$$

$$x^2-e^2x^2+a^2e^2-2aex+y^2 = +a^2-2aex$$

$$x^2(1-e^2)+a^2e^2-a^2+y^2=0$$

$$x^2(1-e^2)+y^2 = a^2-a^2e^2$$

$$x^2(1-e^2)+y^2 = a^2(1-e^2)$$

$$\frac{x^2(1-e^2)}{a^2(1-e^2)}+\frac{y^2}{a^2(1-e^2)}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{a^2(e^2-1)}=1$$

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$$

$e>1$

$a^2(e^2-1)=b^2$

$e^2=\frac{b^2}{a^2}+1$

$e^2=1+\frac{b^2}{a^2}$

$e=\sqrt{1+\frac{b^2}{a^2}}$

Hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Directrix of Hyperbola = $x = \frac{a}{e}$, $x = -\frac{a}{e}$

Focii $(\pm ae, 0)$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

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Exercise 27.1

1. The equation of the directrix of a hyperbola is $x - y + 3 = 0$. Its focus is (-1,1) and eccentricity 3. Find the equation of the hyperbola.

$$\frac{PF}{PL} = e$$

Directrix $x - y + 3 = 0$

$F(-1, 1)$ $\boxed{e = 3}$ $P(x, y)$

$$\frac{\sqrt{(x+1)^2 + (y-1)^2}}{\frac{x - y + 3}{\sqrt{(1)^2 + (-1)^2}}} = e$$

$$\Rightarrow \sqrt{(x+1)^2 + (y-1)^2} = e\left[\frac{x-y+3}{\sqrt{2}}\right]$$

Sq. Both Side

$$(x+1)^2 + (y-1)^2 = \frac{e^2\left[x-y+3\right]^2}{2}$$

$$(x+1)^2+(y-1)^2=\frac{e^2[x-y+3]^2}{2}$$

$$x^2+1+2x+y^2+1-2y=\frac{e^2(x^2+y^2+9-2xy-18y+18x)}{2}$$

$$2(x^2+y^2+2x-2y+2)=9(x^2+y^2+9-2xy-18y+18x)$$

H.W

2.Find the equation of the hyperbola whose

i) Focus is (0,3) , directrix is $x + y - 1 = 0$ and eccentricity=2

Focus (0,3) | Directrix $x+y-1=0$ | $e=2$

Let $P(x,y)$

$$\frac{PF}{PL} = e$$

$$\frac{\sqrt{(x-0)^2+(y-3)^2}}{\frac{x+y-1}{\sqrt{1+1}}} = e$$

$$\sqrt{x^2+(y-3)^2} = e\left[\frac{x+y-1}{\sqrt{2}}\right]$$

Sq. Both Side

$$x^2+(y-3)^2 = e^2\left[\frac{x+y-1}{2}\right]^2$$

$$x^2+y^2+9-6y = e^2\left[\frac{x^2+y^2+1+2xy-2y-2x}{2}\right]$$

$$x^2+y^2+9-6y = \frac{2}{1}\left[\frac{x^2+y^2+1+2xy-2y-2x}{2}\right]$$

$$x^2+y^2+9-6y=2[x^2+y^2+1+2xy-2y-2x]$$

$$x^2+y^2+9-6y=2x^2+2y^2+2+4xy-4y-4x$$

$$-x^2-y^2-7-2y+4x-4xy=0$$

$$-[x^2+y^2+7+2y-4x+4xy]=0$$

$$x^2+y^2+7+2y-4x+4xy=0$$

ii) Focus is (1,1) directrix is $3x + 4y + 8 = 0$ and eccentricity=2

$F(1,1)$ Directrix $\rightarrow 3x+4y+8=0$ $e=2$

$P(x,y)$

$$\frac{PF}{PL} = e$$

$$\frac{\sqrt{(x-1)^2+(y-1)^2}}{\frac{3x+4y+8}{\sqrt{9+16}}} = e$$

$$\sqrt{(x-1)^2+(y-1)^2} = e\left|\frac{3x+4y+8}{\sqrt{25}}\right|$$

Sq. Both Side

$$(x-1)^2+(y-1)^2 = \frac{e^2(3x+4y+8)^2}{25}$$

$$x^2+1-2x+y^2+1-2y = e^2(9x^2+16y^2+64+24xy+64y+48x)/25$$

$$x^2+1-2x+y^2+1-2y=\frac{4\left[9x^2+16y^2+64+24xy+64y+48x\right]}{25}$$

iii) Focus is (1,1) directrix is $2x + y = 1\ and$ eccentricity = $\sqrt{3}$

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iv) Focus is (2,-1), directrix is $2x + 3y = 1$ and eccentricity=2

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v) Focus is (a,0) ,directrix is 2x-y+a=0 eccentricity = $\frac{4}{3}$

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vi) Focus is (2,2) , directrix is x + y = 9 and eccentricity=2

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3. Find the eccentricity ,coordinates of the foci , equations of directrices and length of the latus-rectum of the hyperbola.

i) $9x^2 - 16y^2 = 144$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{\left(\frac{12}{3}\right)^2} - \frac{y^2}{\left(\frac{12}{4}\right)^2}$$

$a > b$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{\frac{144}{16}}{\frac{144}{9}}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Directrix:

$$x = \frac{a}{e}$$

$$x = \frac{\frac{12}{3}}{\frac{5}{4}} = \frac{12}{3} \times \frac{4}{5}$$

$$x = \frac{48}{15}$$

Focus $(\pm ae, 0)$

$$\left(\pm \frac{12}{3} \times \frac{5}{4}, 0\right) = (\pm 5, 0)$$

l. latus rectum $= \frac{2b^2}{a} = \frac{2 \times \frac{144}{16}}{\frac{12}{3}}$

$\frac{18}{4}$ $\frac{9}{2}$

$= \frac{2 \times 12}{16} \times 3$ or 4

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ii) $16x^2 - 9y^2 = -144$

$$-\frac{16x^2}{144} + \frac{9y^2}{144} = 1$$

$$-\frac{x^2}{144/16} + \frac{y^2}{144/9} = 1$$

$a < b$

$$a = \frac{12}{4} \qquad b = \frac{12}{3}$$

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

$$e = \sqrt{1 + \frac{144/16}{144/9}} = \sqrt{\frac{25}{16}} = 5/4$$

Foci $(0, \pm be)$

$$\left(0, \pm \frac{12}{3} \times \frac{5}{4}\right) = (0, \pm 5)$$

Latus rectum $= \frac{2a^2}{b}$

$$= 2 \times \frac{144}{16} \div \frac{12}{3} = \frac{144}{8} \times \frac{3}{12} = \frac{9}{2}$$

$$y = \pm \frac{b}{e} = \pm \frac{12/3}{5/4} = \frac{12}{3} \times \frac{4}{5} = \frac{16}{5}$$

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$$y = \pm \frac{16}{5}$$

$$5y = \pm 16$$

iii) $4x^2 - 3y^2 = 36$

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iv) $3x^2 - y^2 = 4$

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v) $2x^2 - 3y^2 = 5$

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